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**DYNAMIC SCALING IN CELLULAR AUTOMATA  
SIMULATIONS OF DEPOSITION PROCESSES**

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**MARCH 2000**



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13. ABSTRACT (Maximum 200 words) Cellular automata simulations can be used to capture many of the essential features of processes that are difficult to model. They are particularly useful in the study of nonlinear dynamical systems that have complex continuous solutions. In this study, cellular automata models have been employed to investigate the nature of the vapor deposition process by exploring the natural evolution of dynamical dissipative systems using self-organized critical system analysis and spatial scaling measures. A new numerical technique is introduced to analyze the intrinsic structure of evolving surface topography in an effort to better understand the dynamics of the growth processes. This technique is being used to validate the integrity of deposition models through a comparative analysis with experimental data and to determine if a correlation exists between intrinsic surface structure and parameters controlling the deposition process.			
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## INTRODUCTION

The notion of self-organized critical systems has been introduced by Bak *et al.* (ref 1) to provide a consistent explanation for the fractal spatial structures, power law distributions, and flicker noise commonly observed in spatially extended-driven dynamical systems. They have proposed a deposition model with a geometric configuration that evolves toward a critical form. The critical state is an attractor for the dynamics of the system, and the model demonstrates how complex fractal surfaces evolve from simple system dynamics.

## APPROACH

The cellular automata employed in this study are based on a two-dimensional regular lattice of cells using the rules given by Kadanoff *et al.* (ref 2). The basic variable is  $z(i,j)$ , where  $(i,j)$  are the spatial indices and  $z$  represents the height of the lattice. The system is initialized to a planar surface,  $z(i,j) = 0$ . Particles are added at random locations  $z(i,j)$  such that  $z(i,j) \rightarrow z(i,j) + 1$  as long as the sites are stable. After a particle is added and after particle rearrangements, the stability of all lattice sites is determined by  $z(i,j)$  and the nearest neighbor  $z$  values. In the model treated here, a site becomes unstable when

$$\sum_{k=-1}^{k=1} \{z(i,j) - z(i, j-k))H(z(i,j) - z(i, j-k)) + \\ (z(i,j) - z(i-k, j))H(z(i,j) - z(i-k, j))\} > \sigma_c \quad (1)$$

where  $H$  is the Heaviside function and  $\sigma_c$  is the stability parameter. If site  $(i,j)$  becomes unstable,  $z(i,j)$  is reduced by  $\Delta z(i,j)$ , where  $\Delta z(i,j) =$

$$\sum_{k=-1}^{k=1} \{H(z(i,j) - z(i, j-k)) + H(z(i,j) - z(i-k, j))\} \quad (2)$$

The height of the corresponding nearest neighbor cells is incremented by 1 if their height is less than  $z(i,j)$ . If all sites are stable, a new particle is added to the system. The boundaries are periodic and the local rule is applied recursively to the cells whose state is affected by the unstable site. The diffusion process continues until there are no more unstable sites, but no new particles are added until the lattice stabilizes. The number of sites that are changed as the lattice reorganizes after a particle is added is the size of an avalanche, and the total number of particles that have been added to the system is denoted  $n$ .

The avalanches have been extensively explored (refs 1-4), and it has been demonstrated that avalanche sizes have the form of cutoff hyperbolic distributions. However, quantitative analysis of the geometric scaling properties of the evolving automata structures has been limited. One study by Meisel and Johnson (ref 5) has explored the structures in terms of self-similar fractals, and it has been shown that there is a monotonic increase in the Hausdorff-Besokovitch dimension  $D$  with increasing  $\sigma_c$ .

However, the nature of the deposition process and the cellular automata models suggest that the resulting surface morphology belongs to a class of fractals called *surface fractals* (ref 6), which are usually measured in terms of self-affine fractal parameters (ref 7). Self-affine fractals are statistically invariant under anisotropic dilations, whereas self-similar fractals are statistically invariant under isotropic dilations (ref 8). The simulations being investigated in this study pertain to surfaces in  $E^3$  and are characterized by statistical invariance under transformations of the form

$$\{x, y, z\} \rightarrow \{\lambda x, \lambda y, \lambda^H z\} \quad (3)$$

where the Hurst exponent,  $H$ , describes the anisotropic scaling. In a more general case, the  $y$  coordinate would have a coefficient  $\lambda^K$ , and in the case of self-similar fractals,  $H$  would have a value of 1. The invariance expressed in equation (3) implies that any point on a self-affine surface can be represented in the form  $\{r, h(r)\}$ , where the height function  $h(r)$  is a single-valued function of  $r \equiv \{x, y\} \in \Sigma$ . Equation (3) applies over a scaling range that is measured in terms of a parallel correlation length,  $\xi_{//}$ . The parallel correlation length is the distance beyond which there is no correlation in heights between points on the surface. The roughness of the surface ( $\sigma$ ) is defined in terms of the perpendicular correlation length,  $\xi_{\perp}$ , which is related to the rms variations in  $h(r)$ . Assuming a homogenous, self-affine, surface structure, the height correlation function is given by (ref 7)

$$C_h(r) = \left\langle [h(r_0 + r) - h(r_0)]^2 \right\rangle_{r_0} \quad (4)$$

The Hurst exponent  $H$  is defined by the small  $r$  variations in  $C_h(r)$ , i.e.,

$$H = \frac{1}{2} d(\ln(C_h(r))/d(\ln(r)) (r \ll \xi_{//}) \quad (5)$$

where the range of  $r$  corresponding to the linear range in  $C_h(r)$  is referred to as the scaling range. Beyond the scaling range at large  $r$ , the elevations become uncorrelated and  $C_h(r)$  is given by

$$C_h(r) \rightarrow 2\sigma^2 (r > \xi_{//}) \quad (6)$$

where  $\sigma$  is the surface roughness and  $2\sigma^2$  is referred to as the perpendicular correlation length,  $\xi_{\perp}$ . Figure 1 shows the double logarithmic plot of  $C_h(r)$  versus  $r$  for a self-affine fractal model.

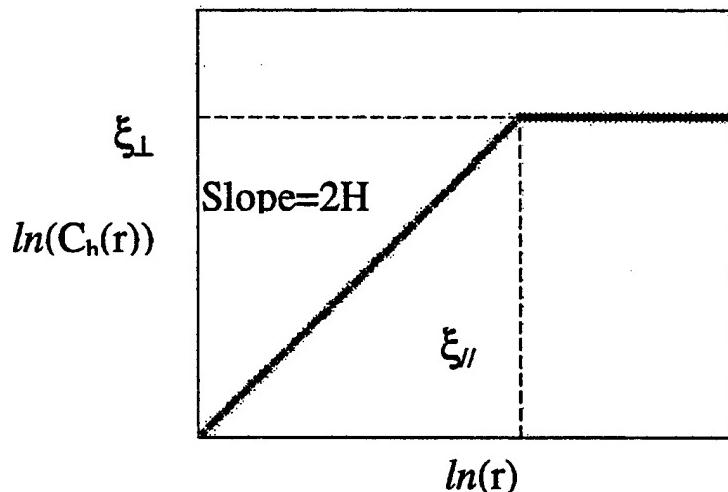


Figure 1.  $C_h(r)$  for a self-affine fractal model.

### MEASURING THE SCALING PARAMETERS

Figures 2a and b show the evolving surface geometry of a cellular automaton model with  $\sigma_c = 9$ . Figure 3 shows the height correlation function corresponding to the surface after  $10^7$  iterations. Although the height correlation function has the functional form of a self-affine fractal model, estimates of  $\xi_{\parallel}$  and  $H$  depend on the range of data used to fit the linear region.

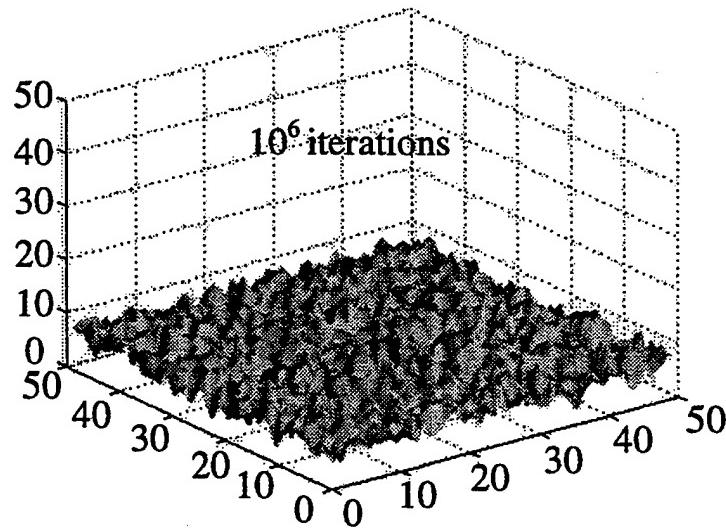


Figure 2a. Evolution of cellular automaton model at  $10^6$  iterations( $\sigma_c = 9$ ).

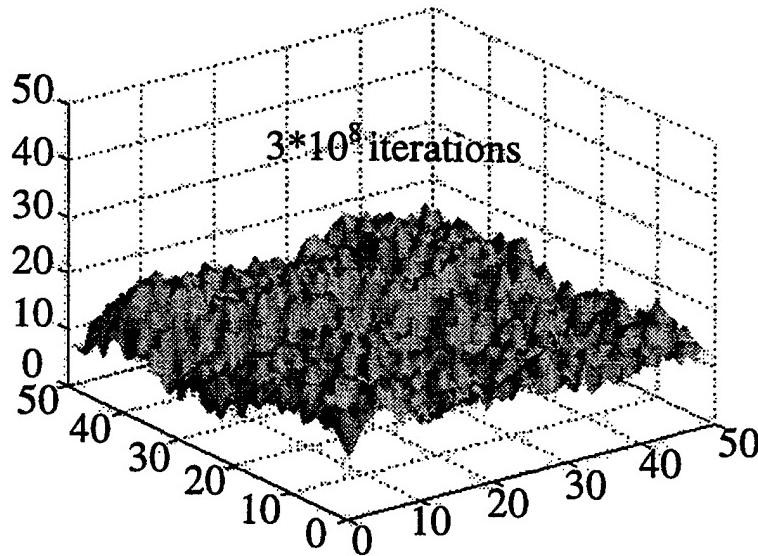


Figure 2b. Evolution of cellular automaton model at  $3 \times 10^8$  iterations ( $\sigma_c = 9$ ).

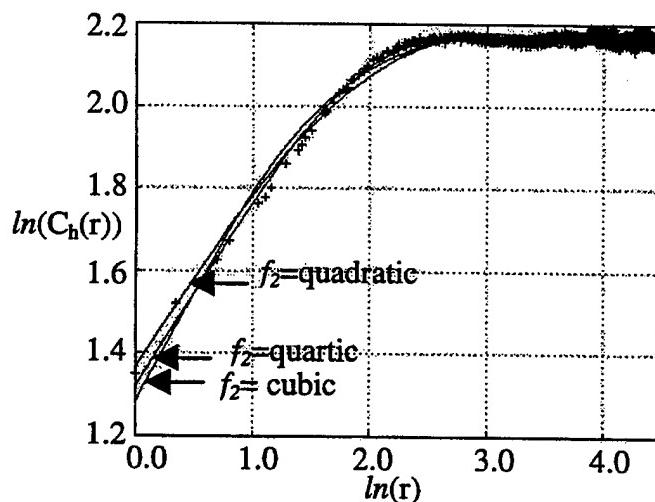


Figure 3.  $C_h(r)$  for  $\sigma_c = 9$  after  $10^7$  iterations.

A new systematic procedure has been developed that measures the scaling parameters by fitting linear and polynomial splines to the data and estimating the values of  $\xi_{//}$  and  $H$  using the data that minimizes the residuals in the fit. The linear range is determined by fitting a straight line,  $f_1$ , to the first  $k$  points and fitting a polynomial,  $f_2$ , to points  $k$  through  $m$ .  $k$  is the index of the data point at which the linear and curved segments meet. It is the variable knot (index) of a linear/curved spline and is a nonlinear parameter determined by exhaustion.  $m$  is the index of the data point at which  $C_h(r)$  approaches a constant ( $\xi_{\perp}$ ). The value of  $m$  is not critical, but should be an estimate of  $\xi_{//}$  for computational efficiency. The coefficients of the polynomials are evaluated with continuity enforced at  $k$ , and the residuals ( $\varepsilon_i$ ) are computed over the entire range. The value of  $k$  for which the residuals are a minimum is selected as the index for the last point of the linear range.  $k$  is computed using a quadratic, cubic, and quartic for  $f_2$ .  $C^1$  and  $C^2$  continuity are enforced for the quartic fit.  $C^2$  continuity is not enforced for the cubic fit to allow some overshoot in the data. The largest of the three values of  $k$  is used to ensure the maximum number of points in the linear region is selected to determine  $H$ . If  $k$  is less than 3, it is assumed that no linear region exists and that the system does not adhere to fractal theory.  $\xi_{//}$  is then determined from the intersection of the linear fit and  $\xi_{\perp}$ . The points used to compute  $\xi_{\perp}$  are at large  $r$  where no correlation is observed. The procedure for the quartic fit is outlined below:

$$f_1(x) = a + b(x - k) \quad (7)$$

$$f_2(x) = c + d(x - k) + e(x - k)^2 + f(x - k)^3 + g(x - k)^4 \quad (8)$$

For  $C^2$  continuity at knot  $k$

$$f_1(k) = f_2(k); f_1'(k) = f_2'(k); f_1''(k) = f_2''(k)$$

and the residuals ( $\varepsilon_i$ ) become

$$\varepsilon_i = a + b p_i - y_i \quad \text{for } i \leq i_k$$

$$\varepsilon_i = a + b p_i + f q_i + g r_i - y_i \quad \text{for } i > i_k$$

where

$$p_i = x_i - k; q_i = (x_i - k)^3; r_i = (x_i - k)^4 \quad (9)$$

Minimizing the sum of the squares of the residuals  $S$

$$S = \sum_{i=0}^m \varepsilon_i^2 = \sum_{LR} \varepsilon_i^2 = \sum_{i=0}^{i_k} \varepsilon_i^2 + \sum_{i=i_{k+1}}^m \varepsilon_i^2 = \sum_L \varepsilon_i^2 + \sum_R \varepsilon_i^2 \quad (10)$$

$$\frac{\partial S}{\partial a} = 0 \Rightarrow a * m + b \sum_{LR} p_i + f \sum_R q_i + g \sum_R r_i = \sum_{LR} y_i \quad (11)$$

$$\frac{\partial S}{\partial b} = 0 \Rightarrow a \sum_{LR} p_i + b \sum_{LR} p_i^2 + f \sum_R p_i q_i + g \sum_R p_i r_i = \sum_{LR} p_i y_i \quad (12)$$

$$\frac{\partial S}{\partial g} = 0 \Rightarrow a \sum_R r_i + b \sum_R p_i r_i + f \sum_R q_i r_i + g \sum_R r_i^2 = \sum_R r_i y_i \quad (13)$$

$$\frac{\partial S}{\partial f} = 0 \Rightarrow a \sum_R q_i + b \sum_R p_i q_i + f \sum_R q_i^2 + g \sum_R q_i r_i = \sum_R q_i y_i \quad (14)$$

The systems of equations (11) through (14) are solved for coefficients  $a, b, f$ , and  $g$  for  $k$  ranging from 2 to  $m-1$ . The data point corresponding to the value of  $k$  for which residuals are a minimum is used as the last point in the linear fit.  $\xi_{//}$  is then determined from the intersection of the linear fit and  $\xi_{\perp}$ . In this investigation,  $k$  was also computed using a cubic ( $g = 0$ ) and quadratic ( $f = g = 0$ ) for  $f_2$ . The largest value of  $k$  evaluated using the three different polynomials for  $f_2$  is used in computing  $H$ . The following system of equations is solved to determine the coefficients of the cubic and quartic fits for  $f_2$ :

Cubic fits:

$$f_2(x) = c + d(x - k) + e(x - k)^2 + f(x - k)^3 \quad (15)$$

$$p_i = x_i - k; q_i = (x_i - k)^2; r_i = (x_i - k)^3 \quad (16)$$

$$\frac{\partial S}{\partial a} = 0 \Rightarrow a * m + b \sum_{LR} p_i + e \sum_R q_i + f \sum_R r_i = \sum_{LR} y_i \quad (17)$$

$$\frac{\partial S}{\partial b} = 0 \Rightarrow a \sum_{LR} p_i + b \sum_{LR} p_i^2 + e \sum_R p_i q_i + f \sum_R p_i r_i = \sum_{LR} p_i y_i \quad (18)$$

$$\frac{\partial S}{\partial e} = 0 \Rightarrow a \sum_R q_i + b \sum_R p_i q_i + e \sum_R q_i^2 + f \sum_R q_i r_i = \sum_R q_i y_i \quad (19)$$

$$\frac{\partial S}{\partial f} = 0 \Rightarrow a \sum_R r_i + b \sum_R p_i r_i + e \sum_R q_i r_i + f \sum_R r_i^2 = \sum_R r_i y_i \quad (20)$$

Quadratic fits:

$$f_2(x) = c + d(x - k) + e(x - k)^2 \quad (21)$$

$$p_i = x_i - k; q_i = (x_i - k)^2 \quad (22)$$

$$\frac{\partial S}{\partial a} = 0 \Rightarrow a * m + b \sum_{LR} p_i + e \sum_R q_i = \sum_{LR} y_i \quad (23)$$

$$\frac{\partial S}{\partial b} = 0 \Rightarrow a \sum_{LR} p_i + b \sum_{LR} p_i^2 + e \sum_R p_i q_i = \sum_{LR} p_i y_i \quad (24)$$

$$\frac{\partial S}{\partial e} = 0 \Rightarrow a \sum_R q_i + b \sum_R p_i q_i + e \sum_R q_i^2 = \sum_R q_i y_i \quad (25)$$

Figure 3 shows the intermediate results with 4 points ( $k = 4$ ) in the linear region. The final value of  $k$  was 15 using a cubic for  $f_2$  which resulted in the scaling parameters of  $H = 0.19$ , and  $\xi_{//} = 8.08$  for  $\xi_{\perp} = 8.76$ .

## DYNAMIC SCALING HYPOTHESIS

Surface structure in deposition processes is time dependent (ref 7), and a measure of evolving morphology can be obtained by applying a generalization of equation (4) as

$$C_h(\mathbf{r}, t) = \left\langle [h(\mathbf{r}_0 + \mathbf{r}, t_0 + t) - h(\mathbf{r}_0, t_0)]^2 \right\rangle_{\mathbf{r}_0, t_0} \quad (26)$$

At small  $t$ , due to the absence of a characteristic time scale (ref 7),  $\xi_{//}$  and  $\xi_{\perp}$  are proportional to powers of time  $t$  and

$$\xi_{//}(t) \propto t^{1/\zeta} \text{ for } t < \tau \quad (27)$$

$$\xi_{\perp}(t) \propto t^B \text{ for } t < \tau \quad (28)$$

$\zeta$  is a conventional exponent for finite size effects, and in general, because of anisotropy in the surface,  $1/\zeta \neq B$ .  $\tau$  is the time at which  $\xi_{\parallel}$  is equal to the size of the sample. In this investigation,  $t$  corresponds to the number of particles added to the system,  $n$ .

## MPI IMPLEMENTATION

The generalized height correlation function,  $C_h(r,t)$ , provides useful information about the dynamics of growth processes, but is numerically expensive. Therefore, a suite of homogenous parallel procedures has been developed to map the solution to a multicomputer platform using an implementation of the *Message Passing Interface* (MPI) called LAM (ref 9) for interprocess communication. LAM is a full implementation of the MPI standard, supports heterogeneous computer networks, and provides direct communication between application processes. It is a node-oriented computing environment that uses a unique identifier assigned to each node as the primary synchronization mechanism. The nodes are usually fully connected (maximum 1-hop distance) since the network is generally a shared resource. In our implementation, a master process assigns equal-sized regions of the lattice to slave processing nodes for computation of partial  $C_h(r,t)$  data. The processing nodes return the results to the master process where  $C_h(r,t)$  is updated and the slave processes assigned new regions to analyze. Although this approach does not exploit the communications advantages of a fully connected network, the load balancing results in a performance improvement directly proportional to the number of workstations. This is likely due to the high computation/communication ratio and the relatively small number of workstations making up the multicomputer.

## RESULTS

The dynamics of the evolving surface morphology of the cellular automata models with  $\sigma_c = 9, 10, 11, 13$ , and  $15$  on a  $256 \times 256$  lattice were characterized in terms of the spatial scaling parameters,  $H$ ,  $\xi_{\parallel}$ , and  $\xi_{\perp}$ . The height correlation function was computed at discrete intervals (iterations), and the largest value of  $k$  resulting in the minimum residuals using three different polynomials for  $f_2$  was computed. The spatial scaling measures were determined for those surfaces for which fractal theory applied ( $k > 2$ ). These values were used to determine the dynamic scaling exponents  $1/\zeta$  and  $B$ . Table 1 shows the measured values of  $H$ ,  $\xi_{\parallel}$ , and  $\xi_{\perp}$  for  $\sigma_c = 9$  at each interval and the degree of the polynomial  $f_2$  that resulted in the largest value of  $k$ . The table shows that  $H$  remains relatively constant as the surface evolves. Results are similar for all other  $\sigma_c$ . In all cases studied, a quartic fit for  $f_2$  resulted in the largest linear region only once.

**Table 1. Spatial Scaling Parameters with  $\sigma_c = 9$**

$N \times 10^6$	$k$	$f_2$	$H$	$\xi_{\parallel}$	$\xi_{\perp}$
1	3	3	0.16	3.13	5.42
3	7	2	0.17	5.37	6.75
5	12	3	0.17	6.69	7.46
7	13	3	0.18	7.32	8.08
10	15	3	0.19	8.08	8.76
30	19	3	0.21	11.59	10.80
50	33	3	0.22	13.20	12.06
70	26	2	0.22	13.87	12.43
100	37	3	0.22	15.33	12.81
300	35	2	0.23	24.78	16.61

The evolving surface morphologies of the cellular automata models adhere well to the dynamic scaling hypothesis as shown in Figure 4. The figure shows typical  $\ln\text{-}\ln$  plots of spatial scaling parameters from which the dynamic scaling exponents are derived. The exponents  $1/\zeta$  and  $B$  are determined from the slope of the straight-line fit using all of the data points. The results for this and the other models are summarized in Table 2. The data show that the value of  $H$  increases with increasing  $\sigma_c$  and at any given point in time, the parallel correlation length decreases with increasing  $\sigma_c$ . This result is expected because as  $\sigma_c$  increases, more particles are required for an avalanche to occur. There is no evident correlation between  $B$  and  $\sigma_c$ .

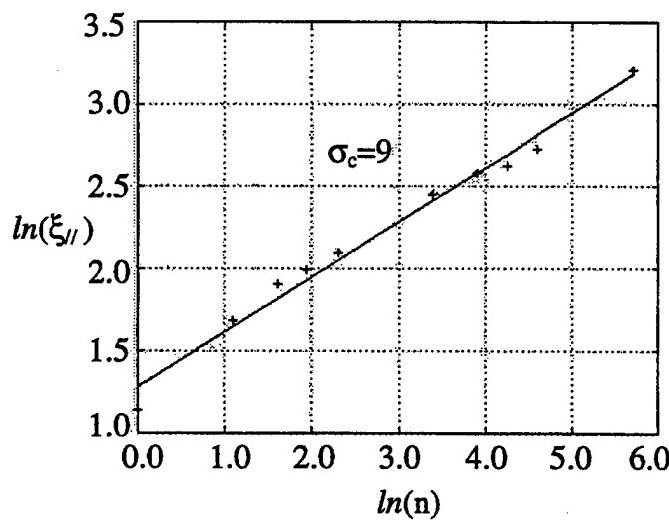


Figure 4. Parallel correlation of length versus time.

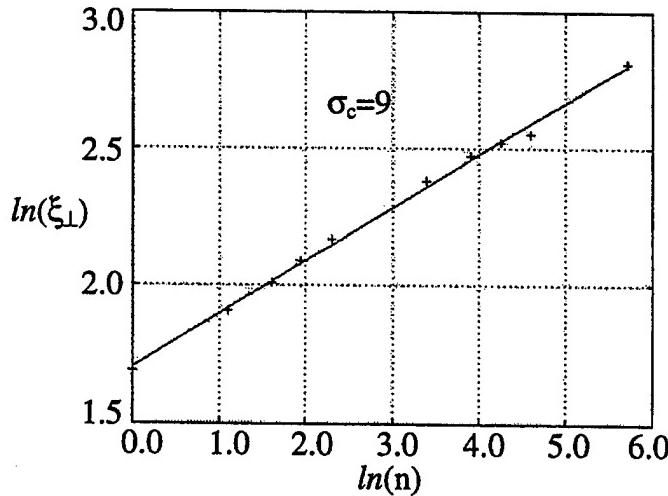


Figure 5. Perpendicular correlation of length versus time.

**Table 2. Dynamic Scaling Parameters**

$\sigma_c$	$\langle H \rangle$	$1/\zeta$	$B$
9	$0.20 \pm 0.03$	0.33	0.19
10	$0.25 \pm 0.04$	0.29	0.23
11	$0.30 \pm 0.03$	0.27	0.23
13	$0.36 \pm 0.03$	0.25	0.24
15	$0.41 \pm 0.01$	0.21	0.20

## SUMMARY

The deposition of adherent, erosion and corrosion-resistant coatings is critical to the performance of many systems. In order to develop optimal coatings, it is important to understand the effects of changing the parameters controlling the deposition/sputtering processes. Models of the deposition processes and quantitative measures of the condition of the substrate surface and evolving coating are, at present, extremely limited. In this study, cellular automata simulations were used to investigate the nature of the vapor deposition process by exploring the natural evolution of dynamical dissipative systems through self-organized critical system analysis and spatial scaling measures. A new numerical technique was developed to analyze the intrinsic structure of evolving surface morphology in an effort to better understand the dynamics of the growth processes. The algorithm is numerically expensive, therefore the computational problem has been mapped to a multicomputer platform using an implementation of MPI call LAM for interprocess communication. Using this approach, the deposition models were shown to agree well with the dynamic scaling theory. The technique is also being used to validate the integrity of other deposition models through a comparative analysis with experimental data, and to determine if a correlation exists between intrinsic surface structure, evolving surface morphology, and parameters controlling the deposition process.

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